Complexity and Retrograde Analysis of the Game Dou Shou Qi

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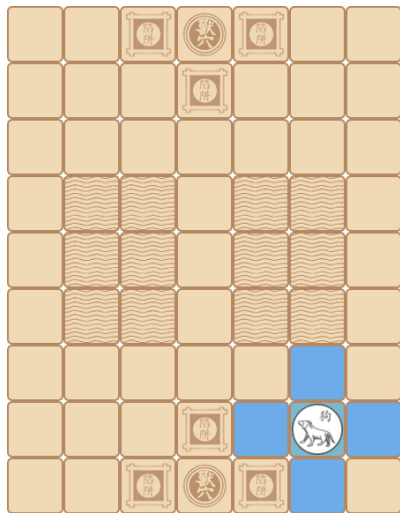November 7, 2013

Dou Shou Qi

Pieces with their strength

Elephant (8) Lion (7)
Tiger (6) Panther (5)
Dog (4) Wolf (3)
Cat (2) Rat (1)

Terrain types

Den — objective square
Traps — reduce enemy strength
Water — "impassable" squares

Dou Shou Qi

Movement
All pieces can move one square either horizontally or vertically
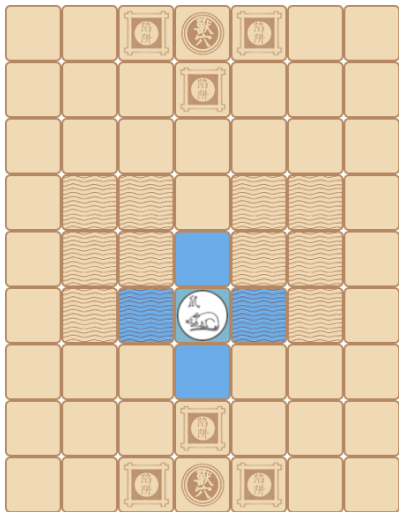
Den
Pieces cannot enter their own den

Traps
Pieces are vulnerable to any enemy piece

Water
Pieces cannot enter the water, . . .

Dou Shou Qi

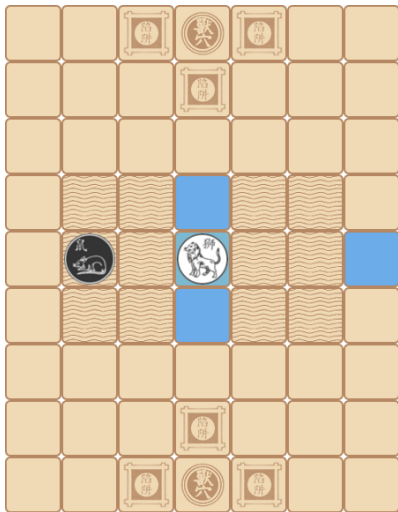## Swimming
Rats can enter the water

Capturing
The Rat (weakest) can capture
the Elephant (strongest)

Exception
Rats cannot capture the elephant
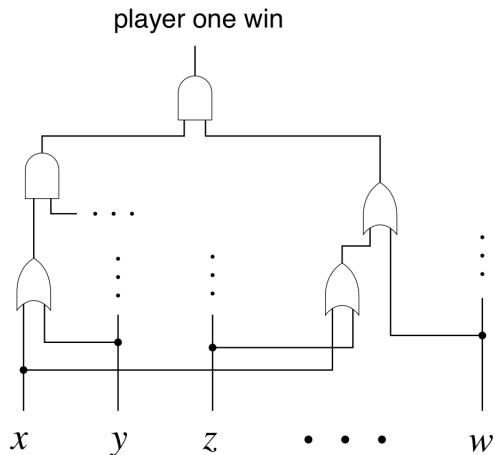from the water

Dou Shou Qi

Leaping

Lions and tigers can leap over the water, both horizontally and vertically
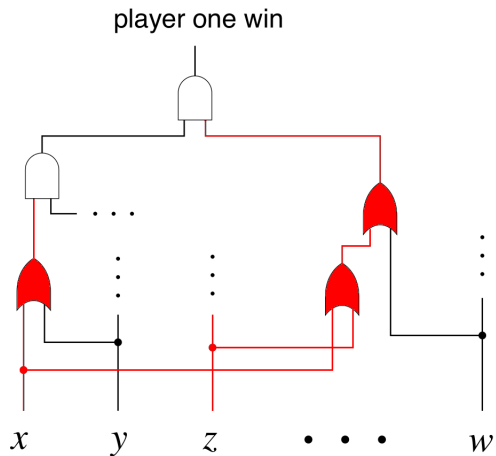
Blocked

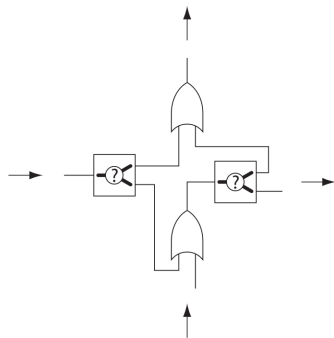Rats in the water block a leap

Circuit Game

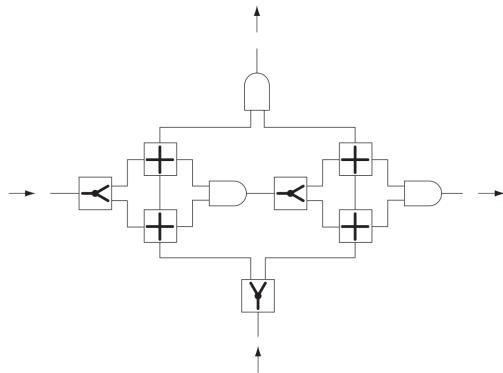CNF formula $(x \vee y) \wedge \ldots \wedge (x \vee z \vee w)$

Circuit Game

CNF formula $(x \vee y) \wedge \ldots \wedge (x \vee z \vee w)$

Planar Circuit Game

(a) Half crossover

(b) Crossover

Reductions

R.A. Hearn

$G_{\mathrm{pos}}(\mathrm{POS\ CNF}) \leq_p \mathrm{Circuit\ Game} \leq_p \mathrm{Planar\ Circuit\ Game}$

Our contribution

$\mathrm{Planar\ Circuit\ Game} \leq_p \mathrm{Dou\ Shou\ Qi}$

Construct gadgets:
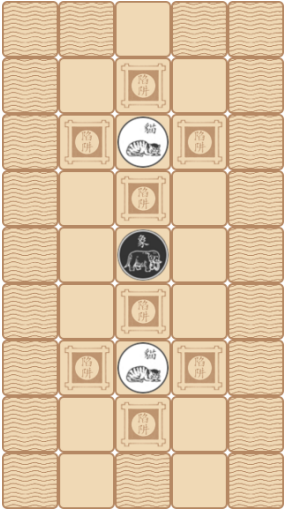
- AND
- OR
- FANOUT
- CHOICE
- VARIABLE

Gadgets

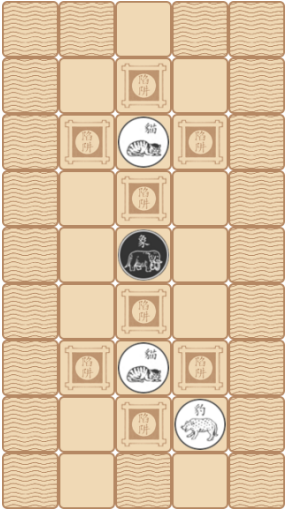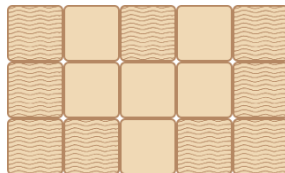
(a) VARIABLE

Gadgets

(b) AND

(c) OR

(d) FANOUT

(e) CHOICE

Unwanted behavior

Problems

- White panthers can go back, effectively reversing the signal in the logic circuit
- Additional panthers can leave the FANOUT gadget through the same exit, effectively doubling the signal in the logic circuit
- Black pieces can escape their gadgets, and possibly destroy other gadgets

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Solution
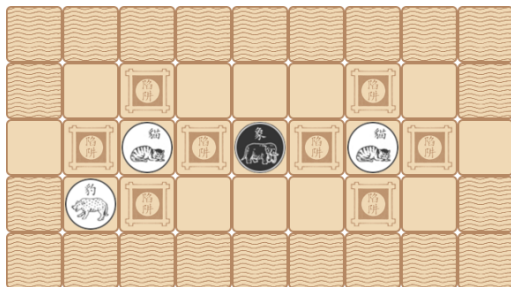
Create additional "protector" gadgets that prevent this behavior
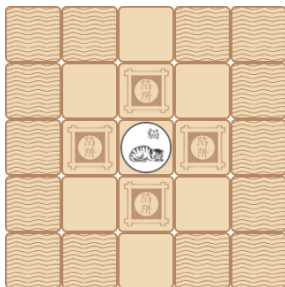
Protector Gadgets

One way gadget

Protector Gadgets

Preventing multiple panthers through one exit

Protector Gadgets

Prevents black pieces from leaving their gadget

PSPACE-hardness

Complexity

Dou Shou Qi is PSPACE-hard

Completeness

Under the assumption of a 50-move rule, PSPACE-completeness can trivially be proven.

Open Problem

We suspect Dou Shou Qi to be EXPTIME-complete, but could not prove it yet.

鬥獸棋 Dou Shou Qi ("Game of Fighting Animals")

bagheera

Moves
1 Rg4 ef7
2 Rf4 dc8
3 Ta2 we8
4 Ea4 cg8
5 Ea5 la8
6 Ta3 lb8

|< < > >|

Configuration: 0f4a7b2g8c3e8f2c8e3c7a3g

Flip board
Rules

visit:
www.liacs.nl/~jvis/doushouqi/

human

Retrograde Analysis

- Endgame tablebase with positions up to four pieces
- Calculating backwards from terminal positions
- Containing almost 10^{10} positions
- Approximately 2% ends in a draw
- Goals:
  - Search for interesting patterns
 - Use it as part of the playing engine

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  - Use it as part of the playing engine
  - Solve Dou Shou Qi in a similar way as Checkers was solved

Retrograde Analysis
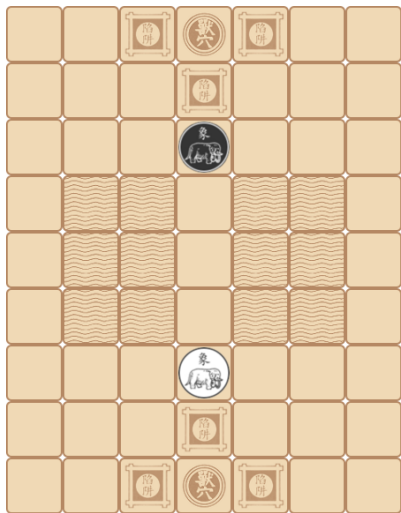
White to play, what is the outcome?

Retrograde Analysis
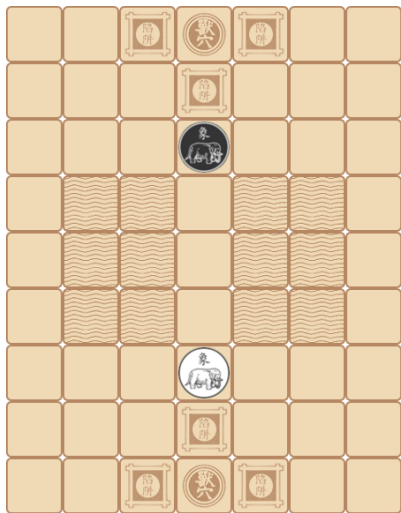
- White loses
- No draws for two equal pieces
- Distance parity is important
- Tigers and Lion can flip parity

White to play, what is the outcome?

Conclusions and Future Work

- Dou Shou Qi is PSPACE-hard, which implies that it is an interesting game to study
- Implementations available[1]: playing engine, web interface and endgame tablebase
- Room for improvement: Can it be proven EXPTIME-complete?
- A reduction on a more regular board
- More interesting patterns can be found in the endgame tablebase

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[1]www.liacs.nl/~jvis/doushouqi/

Questions

Thank you for your attention.