Exercise 2.1: Evolutionary Computation

Consider a population of simple creatures, with a single chromosome of length \( n = 1000 \). Each entry in the chromosome can take four values (A, C, G, T). Assume that the population size is equal to \( M \).

(a) How many possible chromosomes are there?

(b) Assuming that the chromosome length and the population size remain constant, what is the upper limit of the number of different chromosomes evaluated in the course of \( G \) generations?

(c) If the population size is constant and equal to \( 10^{12} \), how large a fraction \( q \) of the total number of chromosomes will be evaluated during \( 10^9 \) generations, assuming that all evaluated chromosomes are different?

Exercise 2.2: Selection Operators

Consider a population consisting of five individuals with the fitness values (before ranking) \( f_1 = 5, f_2 = 7, f_3 = 8, f_4 = 10, f_5 = 15 \). Compute the probability that individual 4 will be selected (in a single selection step) with

(a) roulette wheel selection

(b) tournament selection, with tournament size equal to 2, and the probability of selecting the best individual (in a given tournament) equal to 0.75

(c) roulette wheel selection, based on linearly ranked fitness values, where the lowest fitness value is set to 1 and the highest fitness value set to 10

Exercise 2.3: Travelling Salesman Problem

Write an evolutionary algorithm that searches for the shortest route between \( N \) cities. Use an encoding method such that the chromosomes consist of lists of integers determining the indices of the cities. Examples of five-city paths starting in city 4 are e.g. (4, 3, 1, 2, 5), (4, 1, 5, 2, 3), (4, 5, 1, 2, 3) etc. The first chromosome thus encodes the path 4 \( \rightarrow \) 3 \( \rightarrow \) 1 \( \rightarrow \) 2 \( \rightarrow \) 5 \( \rightarrow \) 4. The fitness should be taken as the inverse of the route
length (calculated using the ordinary cartesian distance measure, i.e. not the city-block distance measure). The program should always generate syntactically correct routes, i.e. routes in which each city is visited once and only once until, in the final step, the tour ends by a return to the starting city. Specialized operators for crossover and mutation are needed in order to ensure that the paths are syntactically correct.

(a) Define a mutation operator for the TSP that maps valid chromosomes (i.e. paths) onto other valid chromosomes.

(b) Define a crossover operator for the TSP that maps valid chromosomes onto other valid chromosomes.

(c) Using the specialized crossover and mutation operators, write an Evolutionary Algorithm that solves the Travelling Salesman Problem.

(d) Apply your program to the TSP given in the file

\[\text{http://ml.informatik.uni-freiburg.de/_media/teaching/ss11/citydata.txt}\]

This file contains a 50 × 2 matrix with the coordinates \((x_i, y_i)\) for city \(i = 1, \ldots, 50\).

What is the shortest path between the cities and how long is it?

**Exercise 2.4: Schema Theorem**

Consider a population containing four individuals with chromosomes 101010, 000111, 010101, and 011011, and fitness values 1, 2, 3 and 4 respectively. In a given selection step, assume that individual 1 (with chromosome 101010) has been selected (using roulette-wheel selection) as the first parent. What is the probability that the schema 10xxxx will be represented in either of the two individuals resulting from the selection of a second parent, followed by crossover? (Crossover may occur, with equal probability, at any of the five available crossover points).

**Exercise 2.5: General Questions**

Discuss possible applications of evolutionary computation. What are the advantages and disadvantages compared to alternative optimization and learning methods? What practical problems arise when using evolutionary algorithms?